How can we detect

Localised Particles in Quantum Field Theory

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DPG Frühjahrstagung Bonn 2025 - AGPhil

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What is Quantum Field Theory (QFT) about?

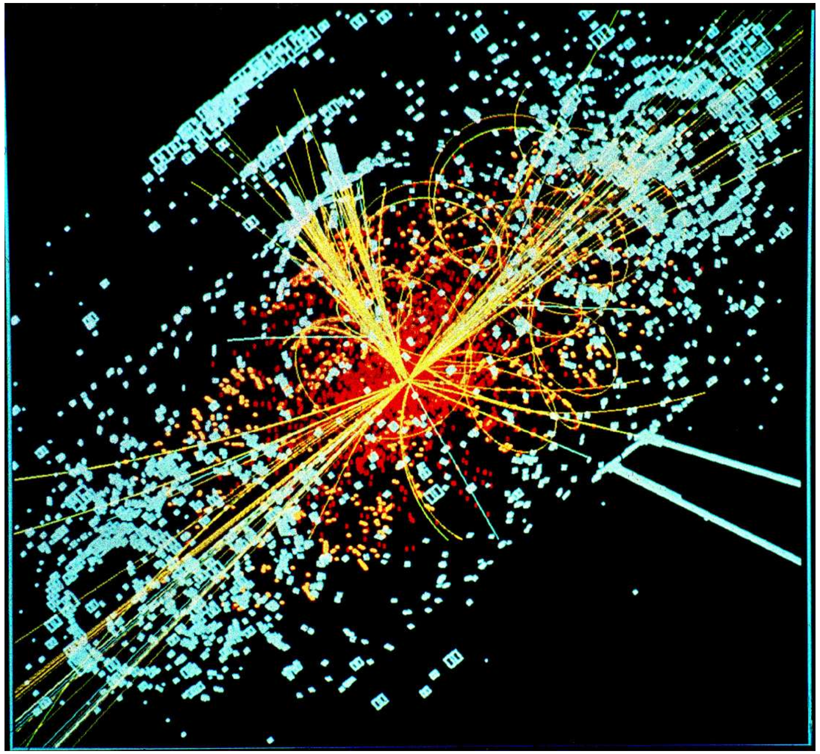
The mathematical formalism does not allow for strictly localised particles

- Halvorson and Clifton 2002: "It is a widespread belief, at least within the physics community, that there is no relativistic quantum theory of (localizable) particles; and, thus, that the only relativistic quantum theory is a theory of fields."

- Kuhlmann 2010: "Although it seems undeniable that modern physics is to a large extent making theories and experiments involving particles it is this very interpretation which has the most fully developed arguments against it."

Intro

But...?



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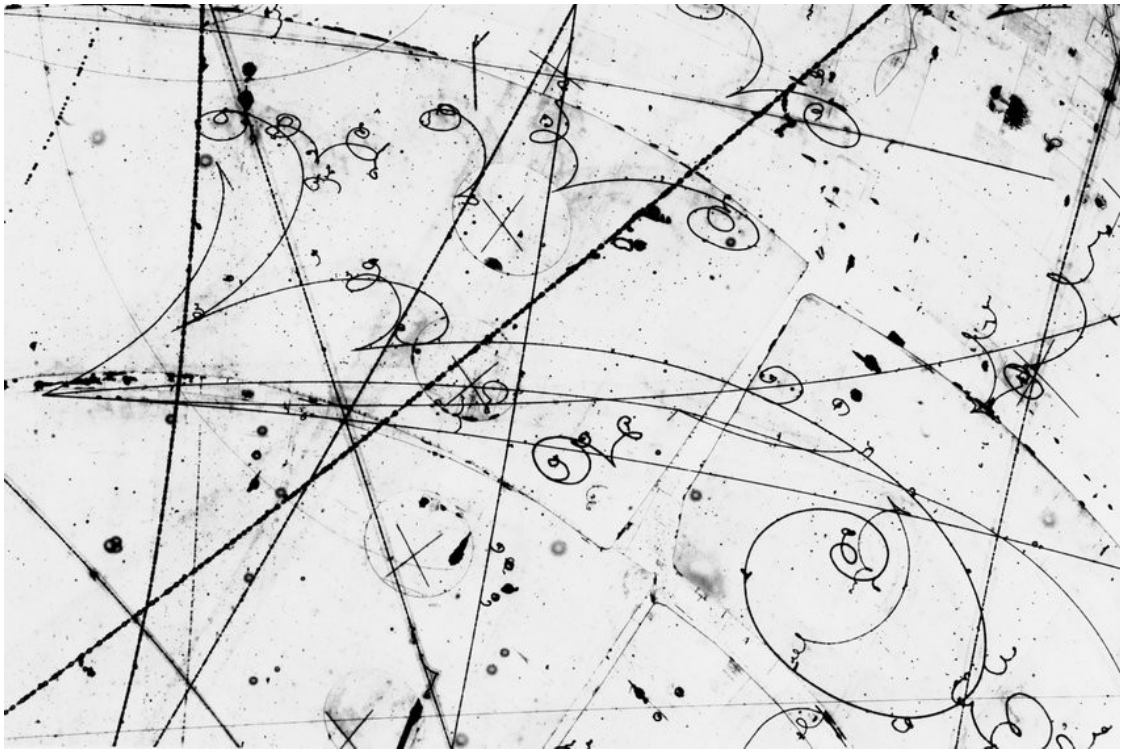

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Intro

Overview

- “Particle Phenomenology without a Particle Ontology” (Arageorgis, Stergiou 2013)
- Several proposals in literature: Wallace, Halvorson and Clifton, Haag, Buchholz and others
- Wallace, Halvorson and Clifton rely on assumption of free theories
- AQFT approach considers scattering theory and concerns asymptotic particle content of theories
- However, it seems that the asymptotic particle content is dependent on a choice of detectors – this is a new form of underdetermination

Intro

Outline

1. No localisation in relativistic QFTs
2. Proposals by Wallace, and Halvorson and Clifton
3. Asymptotic Detector Patterns
4. It’s all about the detectors!

1. No localisation in relativistic QFTs

No-Go Theorems

- Relativistic quantum field theory: Operators are tied to spacetime regions
  - $\phi(x), \pi(x)$ for „Lagrangian QFT“
  - $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$ for „Algebraic QFT“ (with the “quasi-local algebra” $\mathcal{A} = \cup_{\mathcal{O}} \mathcal{A}(\mathcal{O})$)

- No-Go theorems by Malament (1996), Halvorson and Clifton (2002) establish that there cannot be systems of projection operators implementing propositions about particle positions
- Reeh-Schlieder theorem shows that there cannot be non-zero operators localised in a bounded region, that are zero in the vacuum

2. Proposals by Wallace, and Halvorson and Clifton

Wallace: Effective Localisation

- ”Lagrangian QFT”:
  - Basic ontological commitment: expectation values of the field operators $\phi(x), \pi(x)$ tell us about field excitations around x of the system in a state
  - Assume we are dealing with a free theory with a mass gap*

- Effective Localisation:
  - We shall call a state $|\psi\rangle$ ***effectively localised in spatial region Σ_i*** iff for all functions $\hat{f}$ of the field operators we have that

$$\langle\psi|\hat{f}|\psi\rangle - \langle\Omega|\hat{f}|\Omega\rangle\Big|_{x\in\Sigma_i} \gg \langle\psi|\hat{f}|\psi\rangle - \langle\Omega|\hat{f}|\Omega\rangle\Big|_{x\in\Sigma_i'}$$

  i.e. the excitations differ substantially more from the vacuum inside Σ_i than in its space-like complement Σ'_i

* Mass gap: the spectrum of the mass operator $M = \left(P^\mu P_\mu\right)^{\frac{1}{2}}$ is bounded from below by some $\lambda_0 > 0$

Halvorson and Clifton: Almost local observables

- Shift focus to particle detectors:
  - Positive observables $C \in \mathcal{A}(\mathcal{O})$ such that $C|\Omega\rangle = 0$
  - However, this is not possible due to Reeh-Schlieder theorem
  - Solution: choose $C \in \mathcal{A}$ to be ***almost local***, which means they can be approximated by local operators but need not be in any $\mathcal{A}(\mathcal{O})$
    - Almost local operators: $C \in \mathcal{A}$ such that $\forall \varepsilon > 0 \exists C_r \in \mathcal{A}(\mathcal{K}_r) : ||C - C_r|| < \varepsilon$
  - While we *measure* strictly local observables, we can approximate the almost local operators
  - $|\psi\rangle$ is a ***localised particle state*** iff $\exists C \in \mathcal{C}: \omega(C) > 0$

3. Asymptotic Detector Patterns

Particles in „Local Quantum Physics“

- **Particle detectors**: $C \in \mathcal{A}$ positive, almost local and annihilates the vacuum state: $\omega_0(C) = 0$, call the collection of particle detectors $\mathcal{C}$
  - Measurements do correspond to observables
  - Within measurement error bounds there will be enough local and almost local observables; we neither know nor care (Haag, 1996) what the exact correspondence is
- **Detector arrangements**

  Denote by $\alpha_x : \mathcal{A} \to \mathcal{A}$ the automorphism implementing spacetime-translations, where $x \in \mathbb{R}^4$

$$D_n \coloneqq \alpha_{x_1}(C_1) \cdots \alpha_{x_n}(C_n) \text{ for } x_i = (t, \boldsymbol{x_i}) \text{ and } |\boldsymbol{x_i} - \boldsymbol{x_j}| > R$$

Particles in „Local Quantum Physics“

- Call a state ω **at most $\mathbf{n}$-fold localised at time $\mathbf{t}$** iff it cannot trigger any $(n+1)$-fold detector arrangement, i.e. for any choice of the $C_i \in \mathcal{C}$

$$\int_{|x_i - x_j| > R} \omega\big(\alpha_{x_1}(C_1) \cdots \alpha_{x_{n+1}}(C_{n+1})\big)\, d^3\boldsymbol{x_1} \ldots d^3\boldsymbol{x_{n+1}} < \varepsilon,$$

where $\boldsymbol{x_i} = (t, x_i)$ and ε is a chosen background tolerance

- If ω has no component that is less than n-fold localised, we can call ω **exactly n-fold localised**

Particles in „Local Quantum Physics“

- We are now interested in asymptotic particle configurations, i.e. the weak limit points of the state ω for asymptotic times:

$$\lim_{x^0 \to \pm\infty} \omega\big(\alpha_x(C)\big), \qquad \text{for } C \in \mathcal{C}$$

- Haag and Araki (1967) show, assuming a mass gap, that these limits converge weakly to states of the full algebra which are „permanently localised“

Generalisation to Particle Weights

- Buchholz then showed that this can be generalised to theories without a mass gap (e.g. QED) by restricting the class of detectors $\mathcal{C}$ further:
  - Particle detectors are $C \in \mathcal{A}$ that are positive, almost local and annihilate all states with energy below some set threshold δ – call this class of detectors $\mathcal{C}_\delta$
- The resulting $\mathcal{C}_\delta$ is a non-unital subalgebra of $\mathcal{A}$
  (which is also not norm-closed, but closed in a suitable topology generated by a family of semi-norms)
- The limit elements

$$\lim_{x^0 \to \pm\infty} \omega\big(\alpha_x(C)\big) \;,\; \text{ for } C \in \mathcal{C}_\delta$$

cannot be extended to a state on all of $\mathcal{A}$ anymore, the limits are well-defined only on $\mathcal{C}_\delta$. Instead of states, the limit functionals are called **particle weights**

Generalisation to Particle Weights

- Via the GNS construction, using particle weights instead of states, one can construct representations of $\mathcal{C}_\delta$ which can be then extended to representations of $\mathcal{A}$
  (which are unitarily equivalent to the vacuum representation of $\mathcal{A}$, when restricted to subalgebras of finite regions)
- These representations can be attributed a sharp momentum and spin
  - This is similar to constructing single particle Hilbert spaces using representations of the Poincaré group
  - One can then decompose particle weights into **pure particle weights**, again paralleling the decomposition of group representations into irreducible representations

Summary – Asymptotic Particles in AQFT

1. Define a class of particle detectors $\mathcal{C}$
2. Consider equal-time detector arrangements where the distances between detectors are chosen suitably:
3. $D_n \coloneqq \alpha_{x_1}(C_1)\cdots\alpha_{x_n}(C_n)$ for $x_i = (t, \boldsymbol{x_i})$ and $|\boldsymbol{x_i} - \boldsymbol{x_j}| > R$
4. Let $t \to \pm\infty$ and consider the limits of the expectation values of the arrangements
5. This is then the asymptotic particle content of an arbitrary (not necessarily free!) state

Summary – Asymptotic Particles in AQFT

For theories with a mass gap, this recovers the usual picture with an asymptotic space and

- A Fock space representation
- One-particle subspaces given by irreps of the Poincaré group (Wigner's construction)

For theories without a mass gap, we get a particle weight

- A representation via the GNS construction of the particle weight that can be extended to a full algebra-representation
- A decomposition into pure weights, mirroring the decomposition into irreps

4. It’s all about the detectors!

Underdetermination of asymptotic particles

- The asymptotic particle content arising from ω **depend on the choice** of the class of detectors
  - E.g., in non-mass-gap cases: choice of δ determines which “soft particles” show up as proper localisation centres of the asymptotic state
  - If asymptotic configurations are not full states, then expectation values of certain elements of $\mathcal{A}$ might not be well-defined

- The content of the theory captured by the mathematical apparatus, given in terms of $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$ and the state of the system at finite times ω, **does not uniquely fix** the asymptotic particle content
- Conversely, it is unclear whether the asymptotic particle content can fix a unique class of detectors, even given the full theory

4. It’s all about the detectors!

Underdetermination of asymptotic particles

- All this seems to be yet further evidence that QFT cannot be given an underlying ontology in terms of particles – not even in the supposedly ”nice” case of scattering theory!

- Simply assuming free theories in the asymptotic limit (as done by Wallace) obscures this situation

- Unclear how many possible choices of detector subalgebras. If several, what could be criteria to prefer one over the other?

Summary

- No-go theorems show that there cannot be localisable particles in relativistic Quantum Theories
- Several attempts to bring this together with the appearances of particles in HEP experiments
- Construction in AQFT takes scattering theory directly into account
- Asymptotic particles are not uniquely determined by just the net of algebras and state of the field at finite times; a choice of a class of particle detectors is involved

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Thank you!

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